UDC 531.36

ON STABILITY OF EQUILIBRIUM OF NONHOLONOMIC SYSTEMS

PMM Vol. 39, № 6, 1975, pp. 1135 - 1140

A. V. KARAPETIAN
(Moscow)

(Received November 4, 1974)

The problem of stability of motion of nonholonomic systems was first considered by Whittaker in [1], and developed in [2—7] et al. The most general results in investigating the stability of equilibrium of conservative nonholonomic systems and in clarifying the influence of the dissipative forces on this stability, were obtained in [5]. In the present paper we give a further generalization of the results obtained in [5].

1. Let us consider a scleronomous conservative mechanical system constrained by nonholonomic constraints linear in velocities. The generalized velocities $q_1^{\cdot}, \ldots, q_n^{\cdot}$ are assumed connected by $m < n$ nonintegrable relations of the form

$$q_\alpha^{\cdot} = \sum_i d_{\alpha i}(q)\, q_i^{\cdot} \tag{1.1}$$

We write the equations of motion in the Voronets form

$$\frac{d}{dt}\frac{\partial \Theta}{\partial q_i^{\cdot}} = \frac{\partial(\Theta + U)}{\partial q_i} + \sum_\alpha \frac{\partial(\Theta+U)}{\partial q_\alpha} d_{\alpha i} + \tag{1.2}$$

$$\sum_\alpha \Theta_\alpha \sum_j q_j^{\cdot}\left[\frac{\partial d_{\alpha i}}{\partial q_j} - \frac{\partial d_{\alpha j}}{\partial q_i} + \sum_\beta\left(d_{\beta j}\frac{\partial d_{\alpha i}}{\partial q_\beta} - d_{\beta i}\frac{\partial d_{\alpha j}}{\partial q_\beta}\right)\right]$$

$$2\Theta = \sum_{ij} a_{ij}'(q)\, q_i^{\cdot} q_j^{\cdot}$$

Here 2Θ and Θ_α represent the results of eliminating $q_\alpha^{\cdot}$ by means of the relations (1.1) from $2T$ and $\partial T/\partial q_\alpha^{\cdot}$, respectively, where T is the kinetic energy and U is the force function. Let us consider an arbitrary point

$$q_s = q_{s0}, \quad q_s^{\cdot} = 0 \tag{1.3}$$

belonging to the manifold of equilibria

$$\frac{\partial U}{\partial q_i} + \sum_\alpha \frac{\partial U}{\partial q_\alpha} d_{\alpha i} = 0, \quad q_s^{\cdot} = 0 \tag{1.4}$$

of the system (1.1), (1.2) (here and henceforth $i, j = 1, \ldots, n-m$; $\alpha, \beta = n-m+1, \ldots, n$; $s = 1, \ldots, n$) and formulate the problem of stability of the equilibrium (1.3).

2. Let us set

$$q_i = q_{i0} + x_i, \quad q_\alpha = q_{\alpha 0} + x_\alpha + \sum_i d_{\alpha i}(q_{s0})\, x_i$$

in the perturbed motion [5]. Then the equations of perturbed motion assume the form

$$x_\alpha^{\cdot} = \sum_i d_{\alpha i}^*(x)\, x_i^{\cdot} \tag{2.1}$$

$$\frac{d}{dt}\frac{\partial \Theta^*}{\partial x_i^{\cdot}} = \frac{\partial \Theta^*}{\partial x_i} + \sum_\alpha \frac{\partial \Theta^*}{\partial x_\alpha} d_{\alpha i} - \sum_j v_{ij} x_j + \sum_j p_{ij} x_j +$$

$$\sum_\alpha w_{i\alpha}x_\alpha + Q_i(x) + \sum_\alpha \Theta_\alpha^* \sum_j x_j^{\cdot} \nu_{\alpha ij}$$

where
$$v_{ij} = v_{ji}, \quad p_{ij} = -p_{ji}$$

$$v_{ij} + p_{ij} = \left\{\frac{\partial^2 U}{\partial q_j \partial q_i} + \sum_\alpha \left(d_{\alpha j}\frac{\partial^2 U}{\partial q_\alpha \partial q_i} + d_{\alpha i}\frac{\partial^2 U}{\partial q_j \partial q_\alpha}\right) + \sum_{\alpha\beta} d_{\alpha i} d_{\beta j}\frac{\partial^2 U}{\partial q_\beta \partial q_\alpha} + \sum_\alpha \frac{\partial U}{\partial q_\alpha}\left(\frac{\partial d_{\alpha i}}{\partial q_j} + \sum_\beta d_{\beta j}\frac{\partial d_{\alpha i}}{\partial q_\beta}\right)\right\}_0$$

$$X^*(x_i, x_\alpha, x_i^{\cdot}) = X\left(q_{i0} + x_i, q_{\alpha 0} + x_\alpha + \sum_i d_{\alpha i}(q_{s0})x_i, x_i^{\cdot}\right), \quad X = d_{\alpha i}, \Theta, \Theta_\alpha, \nu_{\alpha ij}$$

$\nu_{\alpha ij}$ denote the expressions within the square brackets in (1.2), $Q_i(x)$ are functions the expansion of which in powers of x_s begins with terms of at least second order, $\{\ldots\}_0$ means that the expression contained within the curly brackets is computed at the point $q_s = q_{s0}$, $w_{i\alpha}$ are constants (also dependent on q_{s0}) which will not appear in the conditions of stability or instability and are therefore not given in their explicit form, and $d_{\alpha i}^*(0) = 0$ [5].

We note that $-v_{ij} + p_{ij}$ coincide with the coefficients of the second variation of the force function computed at q_{s0} with (1.1) taken into account.

The characteristic equation of the first approximation system for (2.1) has the form

$$\lambda^m \det \| a_{ij}\lambda^2 + v_{ij} - p_{ij} \| = 0, \quad a_{ij} = a_{ij}'(q_{s0}) \tag{2.2}$$

consequently the problem of stability of solution (1.3) of the system (1.1), (1.2) (or of the solution $x = x^{\cdot} = 0$ of (2.1)) [3] can be reduced to that of investigating the roots of the equation

$$\det \| a_{ij}\lambda^2 + v_{ij} - p_{ij} \| = 0 \tag{2.3}$$

which is the characteristic equation for the system

$$\sum_j a_{ij}y_j^{\cdot\cdot} + \sum_j v_{ij}y_j = \sum_j p_{ij}y_j \tag{2.4}$$

containing the potential forces and the nonconservative eigen forces. The latter will vanish when all $p_{ij} = 0$, i.e. all

$$\left\{\sum_\alpha \frac{\partial U}{\partial q_\alpha}\left(\frac{\partial d_{\alpha i}}{\partial q_j} + \sum_\beta d_{\beta j}\frac{\partial d_{\alpha i}}{\partial q_\beta}\right)\right\}_0 = \left\{\sum_\alpha \frac{\partial U}{\partial q_\alpha}\left(\frac{\partial d_{\alpha j}}{\partial q_i} + \sum_\beta d_{\beta i}\frac{\partial d_{\alpha j}}{\partial q_\beta}\right)\right\}_0 \tag{2.5}$$

In this case the problem of stability of equilibrium of the nonholonomic system is solved simply enough and, in some sense, similarly to the problem of stability of equilibrium of a holonomic system.

3. Theorem 3.1. If $2V = \Sigma v_{ij}x_ix_j$ has a minimum at the point $x = 0$, then with the condition (2.5) holding, the equilibrium (1.3) of the system (1.1), (1.2) is stable in the first approximation.

Proof. We consider

$$W = \Theta^* + V - \Sigma w_{i\beta}x_ix_\beta + \frac{1}{2}c\Sigma x_\beta^2$$

If V has a minimum, then, obviously, we can always choose such $c > 0$ that W is positive-definite in $x_i^{\cdot}$, x_i and x_α. Let us inspect the total time derivative of W, taking due regard of (2.1) (where all $p_{ij} = 0$); after simple transformations we find that $W^{\cdot} = \Sigma Q_j'(x)x_j$, where the expansion of $Q_j'(x)$ in powers of x_s begins with terms of at least second order, i.e. the expansion $W^{\cdot}$ in powers of x_s and $x_i^{\cdot}$ is of at least third order. This proves Theorem 3.1.

Theorem 3.2. If V has no minimum at $x = 0$ and can assume negative values, then with the condition (2.5) holding, the equilibrium (1.3) of the system (1.1), (1.2) is unstable.

The proof is obvious. The equation (2.3) has a root in the right semiplane.

Theorem 3.3. When the conditions of Theorem 3.1 hold, then addition of arbitrarily small dissipative forces with full dissipation in $q_i^{\cdot}$, makes the equilibrium (1.3) of the system (1.1), (1.2), which is stable in the first approximation, Liapunov stable. When the dissipative forces are added, the right-hand sides of (1.2) and of the second group of equations of the system (2.1) must be supplemented by the terms $\partial\Phi / \partial q_i^{\cdot}$ and $\partial\Phi^* / \partial x_i^{\cdot}$, respectively, where $2\Phi = \Sigma f_{ij}'(q) q_i^{\cdot} q_j^{\cdot}$ is the result of eliminating $q_\alpha^{\cdot}$ with the help of (1.1) from the dissipative Rayleigh function $2F = 2F(q_s, q_s^{\cdot})$

$$\Phi^*(x_i, x_\alpha, x_i^{\cdot}) = \Phi(q_{i0} + x_i,\ q_{\alpha 0} + x_\alpha + \sum_i d_{\alpha i}(q_{s0}) x_i,\ x_i^{\cdot})$$

The equations (2.2) and (2.3) now respectively become

$$\begin{aligned} &\lambda^m \det \| a_{ij}\lambda^2 + v_{ij} - p_{ij} + f_{ij}\lambda \| = 0, \quad f_{ij} = f_{ij}'(q_{s0}) \\ &\det \| a_{ij}\lambda^2 + v_{ij} - p_{ij} + f_{ij}\lambda \| = 0 \end{aligned} \tag{3.1}$$

The proof of Theorem 3.3 now follows from the Aiserman-Gantmacher theorem [3], since all roots of (3.1) lie, under the conditions of Theorem 3.3, in the left semiplane.

Theorem 3.4. No dissipative forces can stabilize a position of equilibrium which is unstable under the conditions of Theorem 3.2. The proof is obvious.

Corollary 3.1. Theorems 3.1 — 3.4 are valid for nonholonomic systems with a single independent velocity ($n - m = 1$, as in this case the condition (2.5) holds necessarily).

Corollary 3.2. If V has a minimum and a number of independent velocities is unity ($n - m = 1$), then the equilibrium (1.3) of the system (1.1), (1.2) is stable in any order approximation.

Proof. From the proof of Theorem 3.1 it follows that in this case

$$W^{\cdot} = Q_1'(x_1, x_\alpha) x_1^{\cdot} = [\varphi^{(2)}(x_1, x_\alpha) + \varphi^{(3)}(x_1, x_\alpha) + \ldots] x_1^{\cdot}$$

Obviously, a function $\psi^{(3)}(x_1, x_\alpha)$ such that $\varphi^{(2)} = \partial\psi^{(3)} / \partial x_1$, always exists.

Consider $W_1 = W - \psi^{(3)}$. We then have

$$W_1^{\cdot} = W^{\cdot} - \frac{d\psi^{(3)}}{dt} = \varphi^{(2)} x_1^{\cdot} + [\varphi^{(3)} + \ldots] x_1^{\cdot} - x_1^{\cdot} \frac{\partial\psi^{(3)}}{\partial x_1} -$$
$$\sum_\alpha d_{\alpha 1}^{*} x_1^{\cdot} \frac{\partial\psi^{(3)}}{\partial x_\alpha} = \left[\varphi^{(3)} - \sum_\alpha d_{\alpha 1}^{*} \frac{\partial\psi^{(3)}}{\partial x_\alpha} + \varphi^{(4)} + \ldots\right] x_1 = [\varphi_1^{(3)} + \varphi_1^{(4)} + \ldots] x_1^{\cdot}$$

Thus W_1 begins with the terms of at least fourth order, W_1 is positive definite in x_1, $x_1^{\cdot}$ and x_α (since the quadratic part of W is positive definite and a third order form is added to W). Similarly, we can find that the function $W_2 = W_1 - \psi_1^{(4)}$ ($\varphi_1^{(3)} = \partial\psi^{(4)} / \partial x_1$) is positive definite in x_1, $x_1^{\cdot}$ and x_α, and the expansion of $W_2^{\cdot}$ begins with the terms of fifth order, etc., which proves Corollary 3.2.

4. Let us now consider a general case when not all $p_{ij} = 0$. We at first assume that dissipative forces are absent.

Theorem 4.1. If $\Sigma v_{ij}A_{ij} < 0$, where A_{ij} represents the algebraic complement of the element a_{ij} of the matrix $\{a_{ij}\}$, then the equilibrium (1.3) of the system (1.1), (1.2) is unstable.

The proof follows from the corollary of Theorem 9 of [8].

Corollary 4.1. If V has a maximum at $x = 0$, then the equilibrium (1.3) of the system (1.1), (1.2) is unstable.

Theorem 4.2. If V has a minimum and the roots of the equation $\det \| \mu a_{ij} - v_{ij} \| = 0$ are all equal, then the equilibrium (1.3) of the system (1.1), (1.2) is unstable.

The proof follows from Theorem 4 of [9].

Let us now investigate the effect of the dissipative forces.

Theorem 4.3. If V has a minimum and $f_{ij} = hf_{ij}^{\circ}$, then at sufficiently large h the equilibrium (1.3) is stable.

The proof follows from Theorem 2 of [10] and a theorem of [3].

Note 4.1. Theorem 4.3 shows that an equilibrium unstable under the conditions of Theorem 4.2, can be stabilized by suitable choice of dissipative forces.

Theorem 4.4. If V has a maximum, then the equilibrium (1.3) cannot be stabilized by any dissipative forces.

The proof follows from Theorem 1 of [9].

Note 4.2. An equilibrium unstable under the conditions of Theorem 4.1, can be stabilized by suitable choice of dissipative forces.

Example 4.1.

$$n - m = 2;\ a_{11} = a_{22} = 1,\ a_{12} = 0;\ v_{11} = 1,\ v_{22} = -2,\ v_{12} = 0$$

$$p_{12} = -p_{21} = {}^3/_2,\ f_{11} = 1,\ f_{22} = 3,\ f_{12} = 0$$

In this case Eq. (3.1) assumes the form

$$\begin{vmatrix} \lambda^2 + \lambda + 1 & {}^3/_2 \\ -{}^3/_2 & \lambda^2 + 3\lambda - 2 \end{vmatrix} = 0$$

It follows from the Hurwitz criterion that all roots of this equation lie in the left semiplane.

5. Note 5.1. The results obtained show that the problem of stability of equilibrium of a nonholonomic system can be reduced, under certain conditions, to that of investigating the function V which can be treated as the potential energy of the "reduced" system (2.4), and which coincides with the quadratic part of the function U^* of [5], provided that both parts of the condition (2.5) vanish (for all i, j). However, if we do not limit ourselves to one method of reducing the problem of stability of equilibrium of a nonholonomic system by investigating only the behavior of the function V, then we can easily prove the following assertion:

Theorem 5.1. If $\det \| v_{ij} - p_{ij} \| < 0$, then the equilibrium (1.3) is unstable whether the dissipative forces are present or absent.

In fact, under the condition of Theorem 5.1, the free term of the characteristic equation (3.1) is negative, consequently the equation has at least one positive root (irrespective of whether $f_{ij} = 0$ or $f_{ij} \neq 0$).

N o t e 5 . 2 . Since $(q_{s0}, 0)$ is an arbitrary point of the manifold of equilibria (1.4), the results obtained enable us to investigate the stability of all positions of equilibrium of the nonholonomic system (indeed, v_{ij}, p_{ij} and a_{ij} are all dependent on q_{s0}).

If the first group of equations of the manifold of equilibria (1.4) can be written in the form

$$q_{s0} = z_s(u), \quad u = \{u_1, \ldots, u_l\} \quad (l \geqslant m) \tag{5.1}$$

where u are the parameters of the surface (1.4) [4], then substituting (5.1) into the expressions for v_{ij}, p_{ij} and a_{ij} and applying the results obtained, we can separate on the surface (5.1) the regions of stable or unstable positions of equilibrium.

6. N o t e 6 . 1 . The presence of linear terms in the expansion of the force function U near the point q_{s0} $(\{\partial U / dq_\alpha\}_0 \neq 0)$ and hence in the energy integral, has not led to finding some kind of sufficient conditions of Liapunov stability for the equilibrium of a conservative (dissipative forces absent) system. Nevertheless, this is possible to achieve.

E x a m p l e 6 . 1 . Let us consider a nonholonomic system $(n = 3,\ m = 1)$ representing a particular case of the Bottema example [2] and determined by the kinetic energy $2T = x^{\cdot 2} + y^{\cdot 2} + z^{\cdot 2}$, force function $U = z + {}^1/_2 (ax^2 + by^2)$ and a nonintegrable constraint

$$z^{\cdot} = cyx^{\cdot} \tag{6.1}$$

Obviously, in this case the manifold (1.4) has the form

$$x = y = 0, \ \ z = u; \ \ x^{\cdot} = y^{\cdot} = z^{\cdot} = 0; \ \ u - \text{ is arbitrary} \tag{6.2}$$

Near an arbitrary point of the manifold (6.2), the Voronets equations for the system in question have the form

$$x^{\cdot\cdot}(1 + c^2y^2) + c^2yy^{\cdot}x^{\cdot} - ax = cy, \quad y^{\cdot\cdot} - by = 0 \tag{6.3}$$

Computing $\delta^2 U$ at the points (6.2), we obtain

$$v_{ij} = \left\| \begin{matrix} -a & -c/2 \\ -c/2 & -b \end{matrix} \right\|, \quad p_{ij} = \left\| \begin{matrix} 0 & c/2 \\ -c/2 & 0 \end{matrix} \right\|; \quad a_{ij} = \left\| \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \right\|$$

Then by Theorem 4.1 the equilibria (6.2) are unstable when $-a-b < 0$, i.e. when $a + b > 0$, and by Theorem 5.1 they are unstable when $ab < 0$, i.e. stability is possible only when $a < 0$ and $b < 0$.

Let $a = -\omega^2$ and $b = -\Omega^2$.

We consider any perturbed motion of the system (6.1),(6.3). This motion will satisfy the conditions

$$y = y_0 \cos \Omega t + y_0^{\cdot} \sin \Omega t = A \sin (\Omega t + \varphi) \tag{6.4}$$

$$x^{\cdot\cdot}[1 + A^2c^2 \sin^2(\Omega t + \varphi)] + A^2c^2 \sin(\Omega t + \varphi)\cos(\Omega t + \varphi)\, x^{\cdot} + \omega^2 x = A c \sin(\Omega t + \varphi) \tag{6.5}$$

$$z^{\cdot} = A c \sin (\Omega t + \varphi)\, x^{\cdot} \tag{6.6}$$

where A and φ are expressed in terms of y_0 and $y_0^{\cdot}$, and A is small when y_0 and y_0 are small.

It can be shown that the zero solution of the homogeneous equation corresponding to (6.5) is stable for sufficiently small A and the condition

$$\pi^2\omega^2 < \Omega^2 \tag{6.7}$$

holding (according to the Liapunov criterion), while the inhomogeneous equation has a unique periodic solution the amplitude of which is small when A is small. Consequently, the general solution of (6.5) is small when y_0, $y_0^{\cdot}$, x_0 and $x_0^{\cdot}$ are sufficiently small and condition (6.7) holds. In this case $z^{\cdot}$ are also small, which follows from (6.6), as well as $z - u$ which follows from the existence of the energy integral

$$x^{\cdot 2} + y^{\cdot 2} + z^{\cdot 2} + \omega^2 x^2 + \Omega^2 y^2 + 2(u - z) = 2h$$

and from the smallness of $x^{\cdot}$, $y^{\cdot}$, $z^{\cdot}$, x, y and h under small initial perturbations. It follows therefore that any solution (6.2) of the system (6.1),(6.3) is stable when $b < \pi^2 a < 0$.

When $b < 0$, $a < 0$ and $b > \pi^2 a$, the question remains open.

Investigation of the linear system shows that in this case any equilibrium (6.2) is stable in the first approximation if $a \neq b$, otherwise the equilibrium is unstable.

7. Example 7.1. Let us consider a system consisting of three rough homogeneous cylinders; two identical cylinders each of radius r and mass m roll along an inclined plane, and the third cylinder of radius R and mass M rolls over the first two cylinders. We introduce a set of stationary coordinates on the inclined plane; the x-axis is parallel to the horizontal plane, the y-axis is perpendicular to the x-axis and directed upwards along the inclined plane. We denote the angle of inclination of the plane by α and the angles which the axes of the lower cylinders make with the x-axis, by β and γ ($\beta \neq \gamma$). We introduce the following generalized coordinates: angles φ, φ_1 and φ_2 of natural rotation of the upper cylinder and of two lower cylinders, respectively, the angle θ between the x-axis and the axis of the upper cylinder and the coordinates x and y of the center of mass of the upper cylinder. These six generalized coordinates are connected by four nonintegrable relations [4] which can be reduced to the form

$$x^{\cdot} = R\varphi^{\cdot} \sin\theta + \theta^{\cdot}\, [r(\varphi_1 \sin\gamma - \varphi_2 \sin\beta) / \sin(\gamma - \beta) - y]$$

$$y^{\cdot} = -R\varphi^{\cdot} \cos\theta - \theta^{\cdot}\, [r(\varphi_1 \cos\gamma - \varphi_2 \cos\beta) / \sin(\gamma - \beta) - x]$$

$$\varphi_1^{\cdot} = -\theta^{\cdot}\, \{r[\varphi_1 \sin(\theta - \gamma) - \varphi_2 \sin(\theta - \beta)] / \sin(\gamma - \beta) - x\sin\theta + y\cos\theta\} / [2r\sin(\theta - \beta)]$$

$$\varphi_2^{\cdot} = -\theta^{\cdot}\, \{r[\varphi_1 \sin(\theta - \gamma) - \varphi_2 \sin(\theta - \beta)] / \sin(\gamma - \beta) - x\sin\theta + y\cos\theta\} / [2r\sin(\theta - \gamma)]$$

The force function of the system (with the accuracy to within a constant) is

$$U = -Mgy\sin\alpha - mgr(\varphi_1 \cos\beta + \varphi_2 \cos\gamma)\sin\alpha$$

and the manifold of equilibria (1.4) in this case assumes the form

$$\theta = \pi/2, \quad x = r(\varphi_1 \cos\gamma - \varphi_2 \cos\beta) / \sin(\gamma - \beta)$$

$$\varphi, \varphi_1, \varphi_2, y \text{ — are arbitrary}; \quad \varphi^{\cdot} = \varphi_1^{\cdot} = \varphi_2^{\cdot} = \theta^{\cdot} = x^{\cdot} = y^{\cdot} = 0$$

Any point belonging to this manifold represents an unstable (with or without dissipative forces) position of equilibrium. This follows from Theorem 5.1

$$\det \| v_{ij} - p_{ij} \| = -M(M + m) R^2 g^2 \sin^2\alpha < 0$$

In conclusion the author thanks V. V. Rumiantsev for supervising this paper.

REFERENCES

1. Whittaker, E. T., A Treatise on the Analytical Dynamics. Cambridge Univ. Press, 1927.
2. Bottema, O., On the small vibration of nonholonomic systems. Indag. Math., Vol.

11, f. 4, 1949.
3. Aiserman, M. A. and Gantmacher, F. R., Stabilität der Gleichgewichtslage in einem nichtholonomen System. ZAMM, Bd. 37, Hft. 1/2, 1957,
4. Neimark, Iu. I. and Fufaev, N. A., Dynamics of Nonholonomic Systems. Moscow, "Nauka", 1967.
5. Rumiantsev, V. V., On stability of motion of nonholonomic systems. PMM Vol. 31, № 2, 1967.
6. Nikolenko, I. V., On the influence of nonholonomic constraints on the character of the equilibrium of a system. Zh. Prikl. mekhanika, Vol. 1, № 10, 1965.
7. Nikolenko, I. V., On the stability of equilibrium of nonholonomic Voronets' systems. Ukr. matem. Zh., Vol. 20, № 1, 1968.
8. Lakhadanov, V. M., On the influence of structure of forces on the stability of motion. PMM Vol. 38, № 2, 1974.
9. Agafonov, S. A., On the stability of nonconservative systems. Vestn. Mosk. Univ., Ser. matem. mekhan., № 4, 1972.
10. Karapetian, A. V., On the stability of nonconservative systems. Vestn. Mosk. Univ., Ser. matem. mekhan., № 4, 1975.

Translated by L. K.

UDC 532.593

WAVES IN AN INHOMOGENEOUS FLUID IN THE PRESENCE OF A DOCK

PMM Vol. 39, № 6, 1975, pp. 1140-1142

V. F. VITIUK

(Odessa)

(Received July 9, 1973)

We investigate the propagation of waves generated by oscillations of a section of the bottom of a tank through a two-layer fluid, in the presence of a dock. Wave motions in an inhomogeneous fluid generated by displacement of a section of the bottom of a tank were studied in [1] where the upper surface of the fluid was assumed either to be completely free, or completely covered with ice. In the present paper we use the method given in [2] to investigate a similar problem under the assumption that the fluid surface is partly covered with an immovable rigid plate. The expressions obtained for the velocity potential are used to determine the form of the free surface and of the interface. We show that when the fluid is inhomogeneous, the wave amplitude on the free surface increases, while the presence of a plate reduces the amplitude of the surface waves, as well as of the internal waves in the region between the plate and the oscillating section of the bottom.

An immovable rigid plate occupying the region $y = h,\ x \leqslant -l,\ -\infty < z < \infty$ is situated at the surface of two-layer fluid in which the density and depth of the upper and lower layer are denoted, respectively, by ρ, h and ρ_1, H. The coordinate origin is situated at the interface and the y-axis is directed vertically upwards. The bottom section $y = -H,\ 0 \leqslant x \leqslant a,\ -\infty < z < \infty$ is deformed according to the law